

Higher Maths Booster Questions

Questions 101-200

101. Expand and simplify

$$\sin\left(x - \frac{5\pi}{4}\right) + \cos\left(x + \frac{7\pi}{4}\right)$$

102. a) Show that the function $f(x) = 3x^3 + 6x^2 + 4x + 7$ is never decreasing.

b) Hence, state the x coordinate of its only stationary point.

103. A straight line passes through the point $(-2, 3)$ and makes an angle of $\frac{3\pi}{4}$ radians with the positive direction of the x axis.

Find the equation of this line and determine if it also passes through the point $(5, -4)$.

104. For two right angled triangles. $\sin A = \frac{2}{\sqrt{7}}$ and $\cos B = \frac{1}{\sqrt{3}}$ where A and B are acute angles.

Find the exact value of $\cos(A + B)$.

105. Differentiate

$$\frac{2x^3 - x^2 + 4x}{x\sqrt{x}}$$

106. Two functions, f and g have equations

$$f(x) = 2x \quad g(x) = \sin x^\circ$$

a) Find a formula i) $f(g(x))$ ii) $g(f(x))$

b) Solve $f(g(x)) = 2g(f(x))$ for $0^\circ \leq x \leq 360^\circ$

107. Calculate the rate of change of the function $V(t) = 5t - 7t^2$ when $t = 2$.

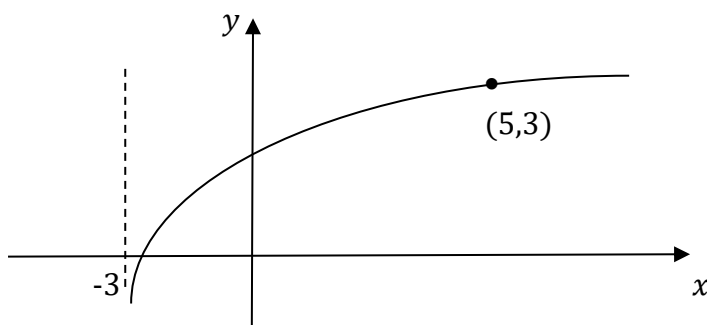
108. Solve

$$\log_3(x^2 + 18) = 2 + \log_3 x \quad x > 0$$

109. Evaluate

$$\int_0^{\frac{2\pi}{3}} 3 \sin 2x \, dx$$

110. The diagram below shows the graph of $y = \log_a(x + b)$. State the values of a and b .



111. The waste produced by a manufacturing company is modelled on the formula

$$W(x) = 8x^2 + \frac{128}{x}$$

Find the value of x which will minimise this waste.

112. State the range of values of x for which $3x^2 + 5x - 12 > 0$.
- 113a. Find the equation of the tangent to the circle $x^2 + y^2 - 6x + 4y + 5 = 0$ at the point $(1, -4)$.
- b. Show that this line is also a tangent to the circle $(x - 5)^2 + (y + 2)^2 = 18$ and state the point of contact.

114. Differentiate

$$\frac{4\sqrt{x}(x^3 - 2x^2)}{x}$$

115. For the recurrence relation, $u_{n+1} = au_n + b$,

$$u_0 = 6, u_1 = 9 \text{ and } u_2 = 10.$$

Find the values for a and b and determine whether this sequence has a limit.

116. Find the equation of the line passing through $(-1, 8)$ which is perpendicular to the line with equation $3x - 5y - 6 = 0$.
117. The points $P(2, 5, 2)$, $Q(6, 2, 4)$ and $R(14, -4, a)$ are collinear.
Find the value of a .

118. A function f has equation $f(x) = 12 + \frac{x}{3}$. Find a formula for the inverse function, $f^{-1}(x)$.

- 119a. Express $4 \sin x^\circ - 5 \cos x^\circ$ in the form $k \cos(x + a)^\circ$.

- b. Hence solve $4 \sin x^\circ - 5 \cos x^\circ + 3 = 0$ for $0^\circ \leq x \leq 360^\circ$

120. Find the rate of change of the function $f(x) = 3 \sin 2x$ when $x = \frac{3\pi}{8}$.

121. Solve

$$\cos 2x + 3 = 2 \sin x \quad 0 \leq x \leq 2\pi$$

122. Circle C_1 has equation $x^2 + y^2 + 12x + 4y + 15 = 0$ and centre A.
Circle C_2 has equation $x^2 + y^2 - 12x + 36y + 135 = 0$ and centre B.

- a. Show clearly that these circles touch externally at a point P.

- bi. State the ratio in which P divides AB.

- bii. Hence, find the coordinates of P.

123. Evaluate

$$\int_1^5 \sqrt{(2x - 1)^3} \, dx \quad x > \frac{1}{2}$$

124. An inverse function, $f^{-1}(x)$, passes through the points $(-3, -2)$ and $(1, 4)$.

State the coordinates of two points the graph of $y = 2f(x + 3)$ passes through.

125. In a body of water, the rate at which ice is melting is being modelled by the equation

$$V = 1635e^{-0.000258t}$$

where V is the volume of ice in tonnes t years after 2020.

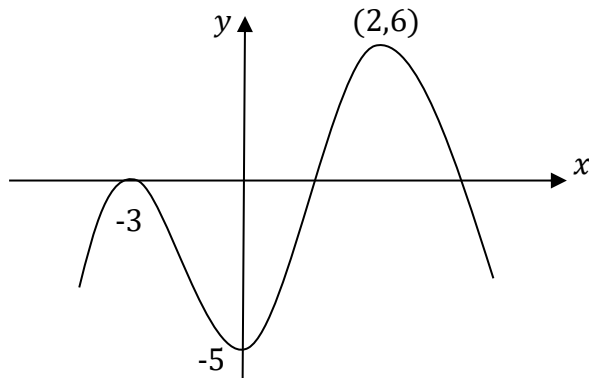
- a. What is the volume of ice initially?

- b. How many years will it take for the ice to be reduced by 30%?

126. Determine whether the function $f(x) = \frac{4}{\sqrt[3]{x}}$ is increasing or decreasing when $x = -8$.

127. The graph of $y = f(x)$ is shown below.

Sketch the graph of $y = f'(x)$.



128. Two functions are defined as

$$f(x) = \sin x + \cos x \quad g(x) = \left(x + \frac{\pi}{3}\right)$$

Express $f(g(x))$ in its simplest form.

129. A line L is perpendicular to the line $x + \sqrt{3}y - 4 = 0$.

State the gradient of L and the angle this line makes with the positive direction of the x axis.

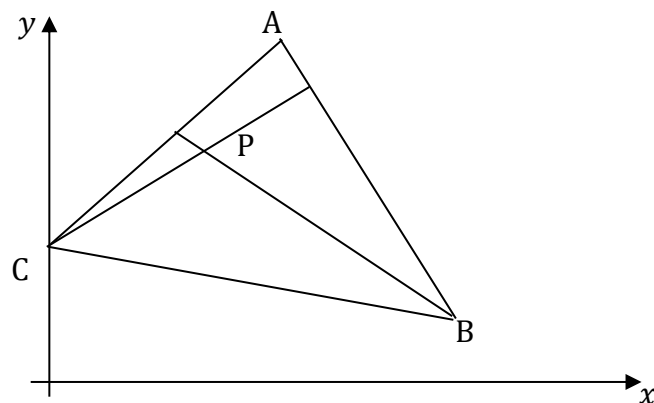
130. State the range of values of k for which the equation $kx^2 - 4x + k = 0$ has two real and distinct roots.

131. A function $g(x)$ is defined on

$$g(x) = \frac{3-x}{8}$$

Find the inverse function, $g^{-1}(x)$.

132. In the diagram below, the vertices of triangle ABC are A(4,8), B(6,2) and C(0,4).



- Find the equation of the median from B.
- Find the equation of the altitude from C.
- Find the coordinates of P, the point of intersection of this median and altitude.

133. a) Show that $(x + 3)$ is a factor of $2x^3 + 7x^2 - 3x - 18$ and fully factorise.

b) Hence solve

$$4x^3 + 14x^2 - 6x - 36 = 0.$$

134. A curve has equation $y = 3x^2 - 4x - 2$

Find the equation of the tangent to the curve where $x = -1$.

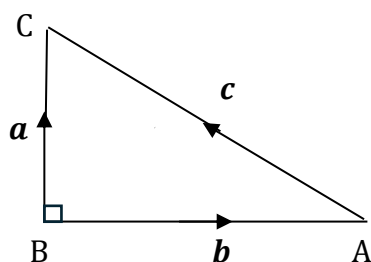
135. A curve has its gradient defined as

$$\frac{dy}{dx} = 3x^2 + 8x$$

The curve is known to pass through the point $(2, 11)$.

Express y in terms of x .

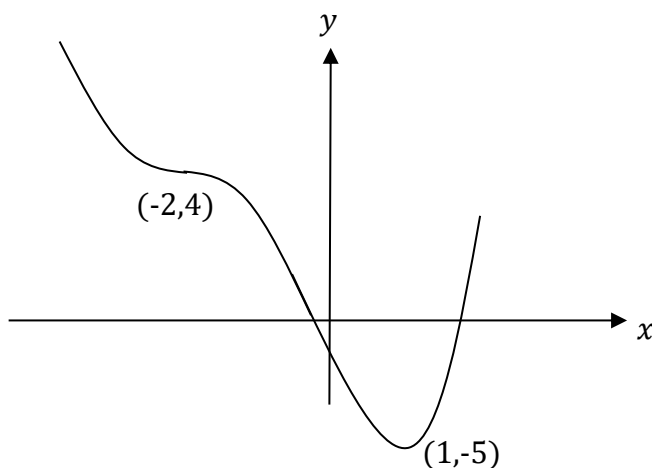
136. Shown below is a right-angled triangle, where $|c| = 4$ and angle BAC is 30°



a) State the values for $|a|$ and $|b|$

b) Hence, evaluate $a \cdot (b + c)$

137. Shown below is the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$.



138. Express $3x^2 + 18x - 20$ in the form $a(x + b)^2 + c$

139. Evaluate

$$\int_1^9 \frac{3}{\sqrt{x}} - \frac{1}{x\sqrt{x}} dx$$

140. Point P has coordinates $(1, -2, 8)$ and point Q has coordinates $(6, -7, 3)$.

R divides PQ in the ratio 2:3.

Determine the coordinates of R.

141. Simplify

$$\log_a 3a^2 - \frac{1}{2} \log_a 9$$

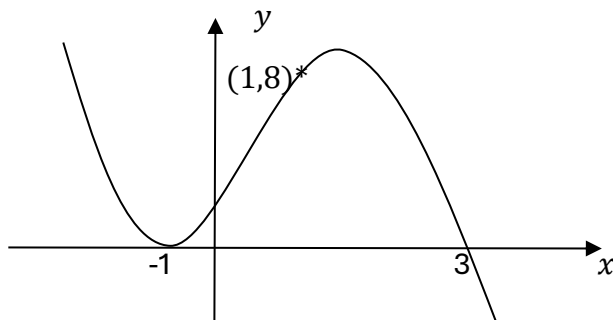
142. Two points A and B have coordinates A(5,-2) and B(-3,-12).

Find the equation of the perpendicular bisector of the line AB.

143. Solve

$$3 \cos x^\circ - 4 \sin 2x^\circ = 0 \quad 0^\circ < x < 360^\circ$$

144. The graph below has equation $y = k(x + a)(x + b)^2$ and passes through the point (1,8).



State the values of k , a and b .

145. Circle C_1 has equation $x^2 + y^2 - 6x + 4y - 5 = 0$ and centre A.

a. Show that the line $y = 7 - x$ is a tangent to this circle and state the point of contact, P.

b i. This line is also a tangent at P to circle C_2 with equation $(x - 18)^2 + (y - 13)^2 = 288$ and centre B.

Show that circles C_1 and C_2 touch externally.

ii. State the ratio in which P divides the line joining AB.

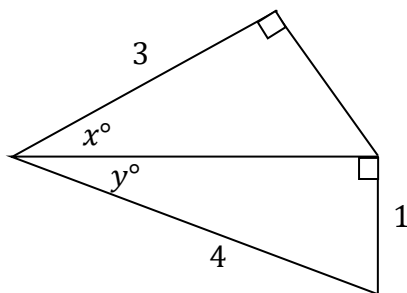
146. The derivative of a function is given by

$$\frac{dy}{dx} = 9x^2 - 4x$$

Its graph is known to intersect with a second graph at the point (-1,4).

Express y in terms of x .

147. Shown below are two right angled triangles.

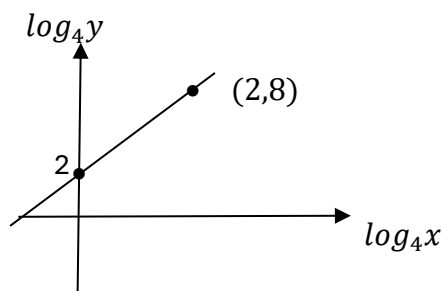


Show that

$$\cos(x + y) = \frac{3\sqrt{15} - \sqrt{6}}{4\sqrt{15}}$$

148. Two graphs have equations $y = x^2 - 3x + 6$ and $y = 4 + x - x^2$.
- Show that these graphs intersect at only one point and state this point of contact.
 - Hence, work out the equation of the common tangent at this point.
149. Express $2 \sin x^\circ - 5 \cos x^\circ$ in the form $k \sin(x + a)^\circ$ $k > 0$.
150. Two variables, x and y , are connected by the equation $y = ax^b$.
The graph of $\log_4 y$ against $\log_4 x$ is shown below.

Establish the values of a and b .



151. Work out

$$\int (2x - 1)^3 dx$$

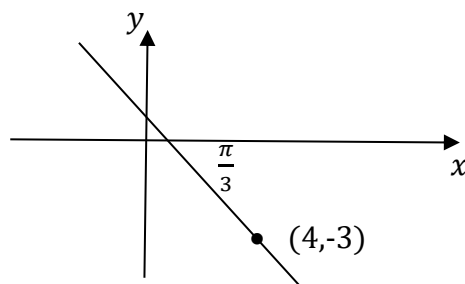
152. Solve

$$\cos 2x - 5 \cos x - 2 = 0 \quad 0 < x < 2\pi$$

153. A pond and a loch will have a chemical added to them to treat a build up of algae.
After being treated, the pond will have 18% less algae, but another 11kg will be added.
- Make up a recurrence relation to model this situation.
A similar treatment is given to the loch.
After the treatment, it will have 22% less algae, but another 14kg will be added.
 - Make up a second recurrence relation to represent this information.
 - Where was the chemical more effective? Show your working.

154. Two functions have equations $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$.
- Work out an expression for $h(x)$ where $h(x) = f(g(x))$.
 - Hence, state the connection between $f(x)$ and $g(x)$.

155. Find the equation of the line shown in the diagram below.

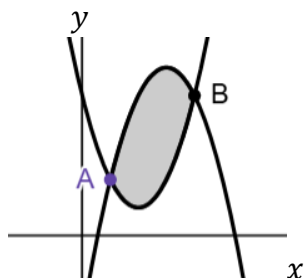


156. Find the rate of change of $y = \cos 3x$ when $x = 2$.

157. Determine whether the vectors $\mathbf{a} = \begin{pmatrix} 2i \\ -5j \\ k \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4i \\ -j \\ 3k \end{pmatrix}$ are perpendicular.

158. Show that $x = -2$ is the only real root of the equation $f(x) = 3x^3 + x^2 - 6x + 8$.

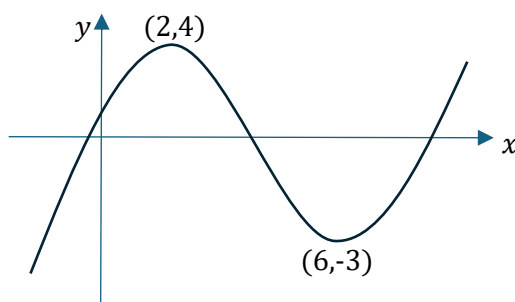
159. In the diagram below, the graphs of $y = x^2 - 4x + 5$ and $y = 6x - x^2 - 3$ intersect at the points A and B.



- Establish the coordinates of the points A and B.
- Hence, work out the area enclosed by both graphs.

160. Show that $f(x) = \frac{2}{3}x^3 - 6x^2 + 18x$ is never decreasing for $x \in \mathbb{R}$

161. The graph of $y = f(x)$ is shown below.



Sketch the graph of $y = 3 - f(x)$.

162. Solve

$$\log_2(x - 1) + \log_2(x + 6) = 3 \quad x > 1$$

- Show that the points A(2,5,6), B(6,15,18) and C(8,20,24) are collinear.
- State the ratio in which B divides AC

164. Differentiate

$$\frac{2x^2+x}{\sqrt[3]{x}} \quad x \neq 0$$

165. Find the coordinates of the stationary points of the function $f(x) = x^3 - 3x^2 - 9x + 14$ and determine their nature.

166. a. Show that

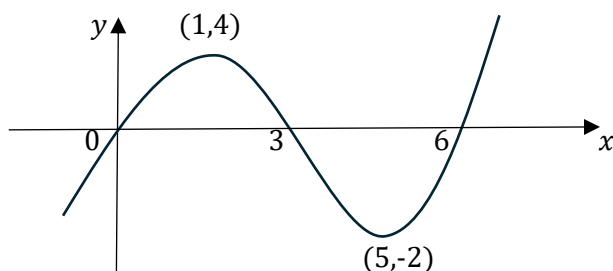
$$\frac{\sin 2x}{\tan x} = 2 \cos^2 x$$

b. Hence, find

$$\int 2 \cos^2 x \tan x \, dx.$$

167. For what value(s) of m does the equation $(m+1)x^2 - 4x + m - 2 = 0$ have real and equal roots?

168. The graph of $y = f(x)$ is shown below.



Make a neat sketch of $y = f'(x)$.

169. A function has its derivative defined as

$$\frac{dy}{dx} = 3x^2 - 8x$$

and is known to pass through the point $(3, 13)$.

Express y in terms of x .

170. Two vectors are defined as $a = \begin{pmatrix} 1 \\ -3 \\ 7 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$.

Calculate the angle between these vectors.

171. A function has equation

$$g(x) = 4 - 3x^2$$

Find a formula for its inverse function and state a suitable domain for this.

172. A circle passes through the points $P(5, 3)$, $Q(7, 1)$ and $R(-1, -3)$.

a) Determine the equation of the perpendicular bisector of QR

b) Determine the equation of the perpendicular bisector of PR

c) Hence, establish the equation of the circle passing through the points P , Q and R .

173. Find the equation of the tangent to the curve $y = 2x^3 + 4x$ when $x = -1$.

174. A sequence is based on the recurrence relation

$$u_{n+1} = au_n + b \quad u_3 = 8 \quad u_4 = 14 \quad u_5 = 17$$

a. Establish the values for a and b .

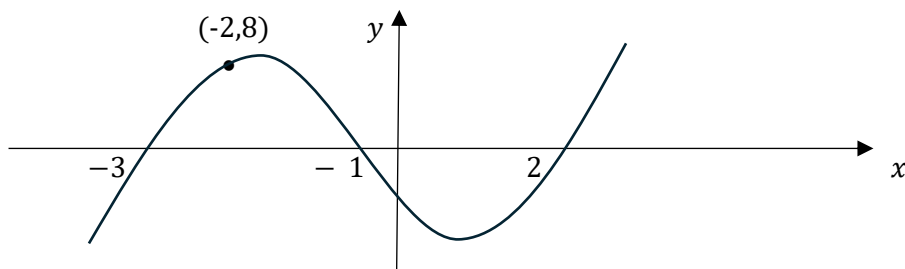
b. Explain why this sequence has a limit and calculate this limit.

175. Show that the line $y = x + 2$ is a tangent to the curve with equation $y = 2x^2 - 3x + 4$ and state the point of contact.

176. Solve

$$\cos 2x + 3 \sin x + 4 = 0 \quad 0 \leq x \leq 2\pi$$

177. The graph of $y = k(x + a)(x + b)(x + c)$ is shown below. State the values of a, b, c and k .



178. Work out

$$\int_1^4 \frac{4\sqrt{x} + 3x^2}{\sqrt{x^5}} dx$$

179a. Differentiate

$$f(x) = \frac{1}{(2x-7)^3}$$

b. Hence find $f'(3)$.

180. Show that

$$\sin 2x + 2 \cos x \tan x = 2 \sin x (\cos x + 1)$$

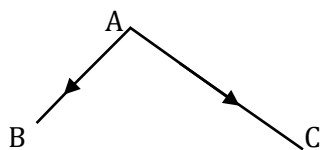
181. a. Find the coordinates of the stationary points on the curve $f(x) = x^3 - 6x^2 + 11$ and determine their nature

b. Hence, work out the maximum and minimum value of $f(x)$ in the interval $-3 \leq x \leq 5$

182. a. Express $5 \sin x^\circ + 3 \cos x^\circ$ in the form $k \sin(x - a)^\circ$.

b. Hence, solve $5 \sin x^\circ + 3 \cos x^\circ - 4 = 0$ for $0^\circ < x < 360^\circ$.

183. In the diagram below, A is (5,-3, 8), B is (1,-2,9) and C is (7,3,4).



a. Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form

b. Hence, calculate the size of angle BAC

184. Evaluate

$$\log_6 4 + \log_6 18 - \log_6 2$$

185. The rate of change of a function is given by $\frac{dv}{dt} = 6t - 4$. It is known that $v = 14$ when $t = 2$. Express v in terms of t .

186. A straight line has equation $\sqrt{3}x + y - 2 = 0$.

Work out the angle this line makes with the positive direction of the x axis.

187. For two right-angled triangles, $\tan A = \frac{4}{\sqrt{2}}$ and $\cos B = \frac{2}{\sqrt{10}}$, where A and B are acute angles.

Show that the exact value of $\sin(A + B) = \frac{4 + \sqrt{3}}{3\sqrt{5}}$.

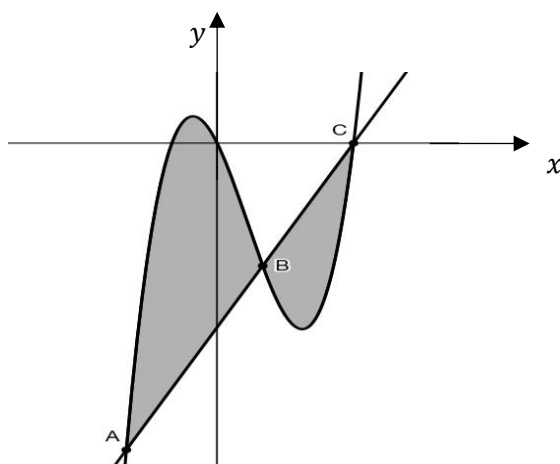
188. A new business was launched in 2022. Its value at any time is based on the model

$$V(t) = 196e^{0.0879t}$$

where V is the value of the business in £million t years after 2022.

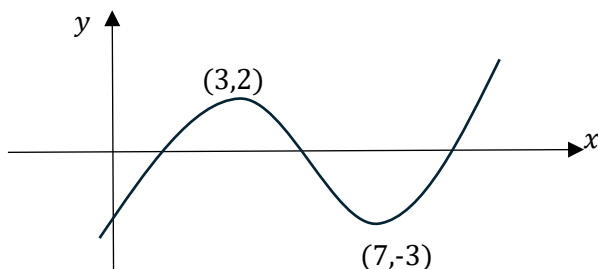
- What was the value of the business in 2022?
- In which year will the business have doubled in value?

189. The graph below shows the curve with equation $y = x^3 - 2x^2 - 3x$ and the straight line $y = 2x - 6$.



The line and curve intersect at the points A, B and C.

- Given that the x coordinate of A is -2 , establish the x coordinates of B and C.
 - Hence, calculate the total shaded area.
190. For what value(s) of k does the equation $y = (k - 3)x^2 + kx + 3 = 0$ have real and equal roots?
191. A function has equation $f(x) = 3x^3 - 5x^2 + 7$. Determine whether the function is increasing or decreasing when $x = -1$.
192. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$.



193. Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 3x + 5$ where $x = 1$.
194. Determine

$$\int 3 \cos 2x + \sin 2x \, dx$$

195. The graph of $y = \log_3(x + 9)$ crosses the x axis at the point A and the y axis at the point B. State the coordinates of points A and B.

196. The total surface area of a solid in square centimetres is given by

$$A(x) = 4x^2 + \frac{512}{x}$$

- Find the value of x which minimises this surface area.
- Calculate this minimum area.

197. a. Express $6 \sin x^\circ - 3 \cos x^\circ$ in the form $k \cos(x - a)^\circ$

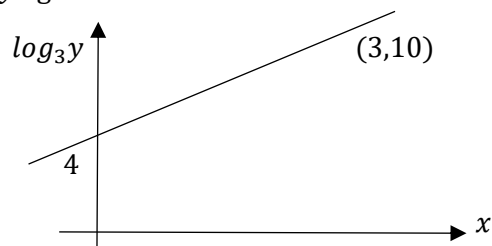
b. Hence solve $6 \sin x^\circ - 3 \cos x^\circ = 5$ $0^\circ \leq x \leq 360^\circ$

198. Points A, B and C have coordinates A (4,1,3) , B(6,5,6) and C(12,17,15).

State the ratio in which B divides AC.

199. Two variables, x and y are connected by the $y = ab^x$

The graph of $\log_3 y$ against x is shown below.



Determine the values for a and b .

200. Work out

$$\int \frac{6}{\sqrt{(3x-7)}} dx.$$