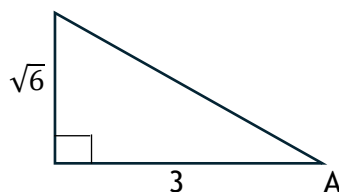


Higher Maths

Booster Questions 1 - 100

1. A function has equation $f(x) = \frac{2}{x} + 5x^2$. Find $f'(-1)$.
2. AB is a straight line where A is (2,-5) and B is (4,-1). Find the equation of the perpendicular bisector of the line AB.
3. A straight line makes an angle of 135° with the positive direction of the x axis and passes through the point (1,6). Find the equation of this line.
4. Evaluate $\log_3 6 - \log_3 2 + \log_3 81$.
5. A function exists where $g(x) = 4 - \frac{3}{x}$. Find the formula for the inverse function, $g^{-1}(x)$.
6. Two functions f and g have equations $f(x) = 3x^2 - 4$ and $g(x) = x - 3$. Find an expression for the function h , where $h(x) = f(g(x))$.
7. Show that the circles with equations $x^2 + y^2 - 8x + 2y - 8 = 0$ and $x^2 + y^2 + 4x + 6y + 4 = 0$ intersect.
8. Show that $(x + 4)$ is a factor of the function with equation $f(x) = 2x^3 + 7x^2 - 5x - 4$ and fully factorise $f(x)$.
9. A triangle has vertices A(3,-4), B(1,-2) and C(-5,8). Find the equation of the altitude from A.
10. Find the rate of change of $y = 2 \sin 3x$ when $x = \frac{\pi}{4}$.
11. A circle has equation $x^2 + y^2 - 6x + 4y - 12 = 0$.
 - a) State the centre and radius of the circle.
 - b) Find the equation of the tangent to the circle at the point (6,2).
12. Find the derivative of the equation $y = \frac{3}{\sqrt{x^3}} + 5\sqrt{x}$.
13. An element decays at the rate of $A = 174e^{-0.0000563t}$ where A is its weight in mg after t hours.
 - a) What is the initial weight of the element?
 - b) Work out the half-life of the element (how long it takes for the element to half its initial weight)
14. Express $3x^2 - 12x + 29$ in the form $a(x + b)^2 + c$.
15. A function exists where $f(x) = \frac{3}{\sqrt{2-x}}$. State the range of values for which $f(x)$ is undefined.
16. Given the right angled triangle below, state the exact values of $\sin 2A$ and $\cos 2A$.



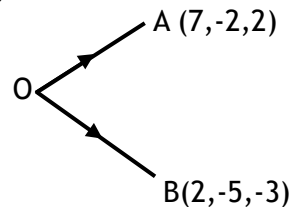
17. Solve $\cos^2 x - \sin^2 x = \sin x$ for $0^\circ < x < 360^\circ$

- 18a. A pesticide, KillBee, is sprayed onto roses to prevent an infestation of bees. After its first spray, it destroys 72% of the bees, but another 85 attack the roses.
Establish a recurrence relation to represent this.
- 18b. Another pesticide comes onto the market, BeeKill. It destroys 84% of the bees, but another 94 attack the roses.
Establish a second recurrence relation to represent this.
- 18c. Which pesticide is more effective in the long term?

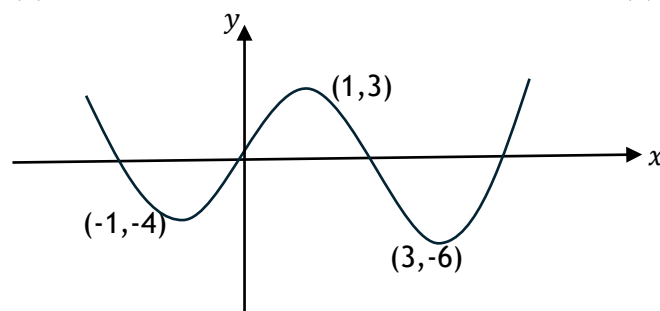
19. Evaluate

$$\int_1^2 \frac{1}{2}x^2 + 6x - 3 \, dx$$

20. Calculate the rate of change of the function $D(t) = 7t^2 - 16t$ when $t = 4$
21. Find the equation of the straight line which passes through the point $(8, -4)$ and makes an angle of 120° with the positive direction of the x axis.
- 22a Simplify $f(x) = \log_a(x^2 - 5x - 24) - \log_a(x + 3)$
- 22b. For what value(s) of x is $f(x)$ undefined?
23. A triangle has vertices $P(-2, 7)$, $Q(8, 3)$ and $R(2, -5)$. Find the equation of the median from P .
- 24a. Express $3 \cos x + 5 \sin x$ in the form $k \sin(x + a)^\circ$ for $0^\circ \leq x < 360^\circ$
- 24b. State the minimum value of $3 \cos x + 5 \sin x$ and the corresponding value of x at which this value occurs.
25. Calculate the angle between the two vectors shown in the diagram below



- 26a. Show that the points $A(3, -5)$, $B(8, -1)$ and $C(18, 7)$ are collinear
- 26b. State the ratio in which B divides AC .
27. Show the exact value of $\frac{\cos 30^\circ + \sin 330^\circ}{\tan 315^\circ} = \frac{1 - \sqrt{3}}{2}$.
28. Solve the inequality $3x^2 + 11x - 20 > 0$.
29. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$.



30a. A function has equation $f(x) = 4x^2 + 1$. Find a formula for the inverse function $f^{-1}(x)$.

30b. State the values of x for which $f^{-1}(x)$ is undefined

30c. Make a neat sketch of the inverse function.

31. Two functions, f and g are defined below.

$$f(x) = x^2 - 3x + 1 \quad g(x) = 2x + 5$$

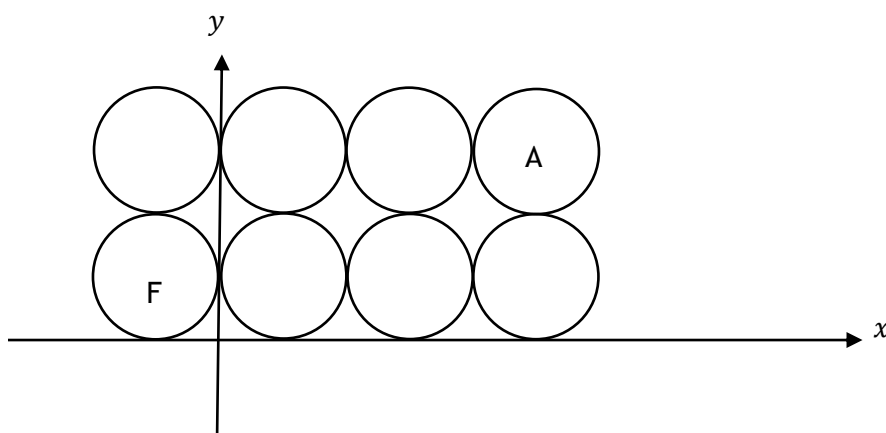
Find expressions for $f(g(x))$ and $g(f(x))$

32. Find

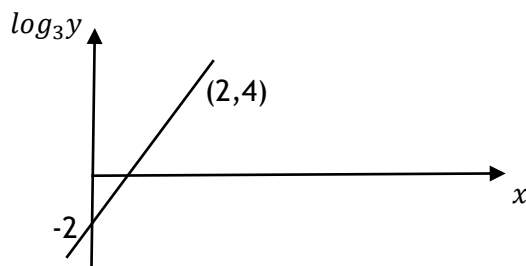
$$\int \frac{5}{\sqrt{6x-1}} dx \quad x > \frac{1}{6}$$

33. The diagram below shows 8 identical circles.

The equation of circle A is $x^2 + y^2 - 30x - 18y + 297 = 0$
Work out the equation of circle F.



34. Two variables, x and y , are connected by the equation $y = ab^x$
The graph of $\log_3 y$ against x is shown below.



Find the values of a and b .

35. State whether the function $f(x) = 2x^3 + 7x^2 - 11x + 2$ is increasing or decreasing at $x = -1$.

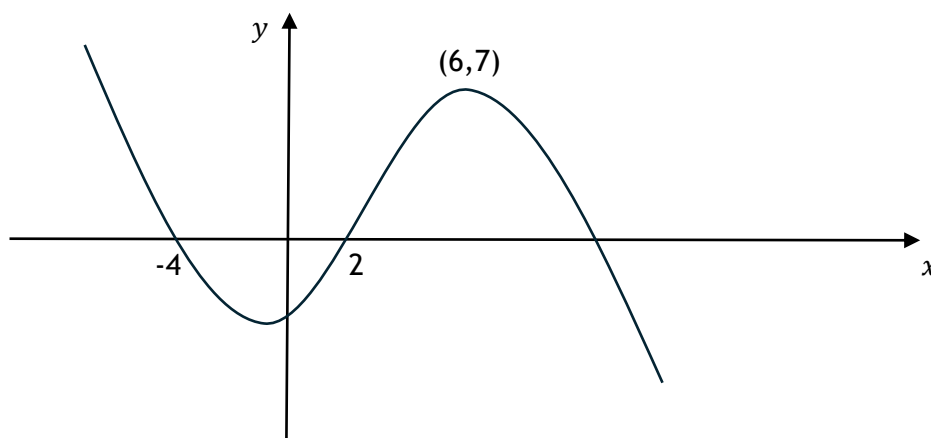
36. The surface area of a solid is given by the formula

$$A(x) = 6x^2 + \frac{96}{x}$$

Find the value of x which minimises this area.

37. State the range of values of p for which the quadratic equation $x^2 + (p + 2)x + 9$ has no real roots.

38. The graph of $y = f(x)$ is shown below. Sketch and annotate the graph of $f(2x) - 3$.



39. Solve

$$\log_4(x^2 - x - 6) - \log_4(x + 2) = 2$$

40. Given that $(x + 4)$ is a factor of the function $f(x) = 2x^4 + x^3 - 17x^2 + 34x - 40$, state $f(x)$ in its fully factorised form and show clearly that there are no other factors.

41. Solve

$$2 \sin^2 x + \cos x = 1 \quad 0 < x < 2\pi$$

42. Determine whether the line passing through the points $(4, 6)$ and $(-4, 5)$ is perpendicular to the line with equation $\frac{1}{4}x + 2y - 6 = 0$

- 43a. A patient is administered 25mg of medication at 10 00 am. Each hour, 30% of this medication passes through its bloodstream and another 12mg is injected. Write down a recurrence relationship for this and work out how much medication remains in the bloodstream after 3 hours.

- 43b. It is unsafe to continue with this level of medication if it exceeds 42mg. Should this treatment continue? Justify your answer.

44. A industrial tank is full of water. As it is emptied, the rate of change of its depth is given by

$$\frac{dD}{dt} = \frac{1}{3}t^2 - 6t + k$$

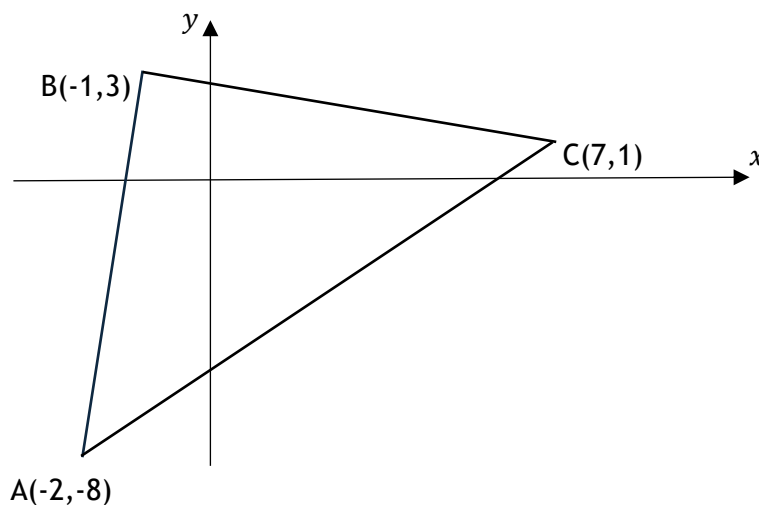
D is depth of water remaining after a time of t seconds and k is a constant. Initially, the depth of water in the tank is 170cm. After 6 seconds, the depth is 152 cm.

Express D in terms of t .

45. Show that the line with equation $y = 7x - 1$ is a tangent to the curve $y = 2x^2 + 3x + 1$ and find the point of contact.

46. Two vectors, $\mathbf{a}$ and $\mathbf{b}$, are perpendicular. Vector $\mathbf{a}$ has components $\begin{pmatrix} 4 \\ -4 \\ 3p \end{pmatrix}$ and vector $\mathbf{b}$ has components $\begin{pmatrix} p+4 \\ -6 \\ 2 \end{pmatrix}$. Find the value of p .

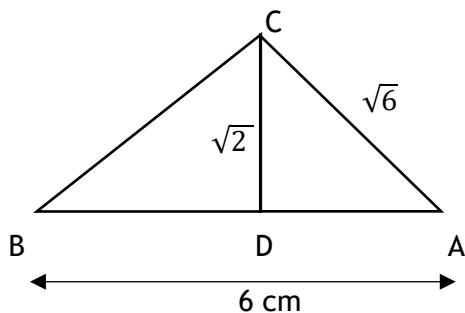
47. In the diagram below, triangle ABC has vertices A(-2,-8), B(-1,3) and C(7,1) as shown.



- Work out the equation of the median from A
 - Work out the equation of the altitude from B.
 - Establish the coordinates of the point of intersection of this median and this altitude.
48. a) Express $5 \cos x^\circ - 2 \sin x^\circ$ in the form $k \sin(x - a)^\circ$.
 b) Hence, solve $5 \cos x^\circ - 2 \sin x^\circ = 4 \quad 0^\circ \leq x \leq 360^\circ$

49. In the diagram shown,

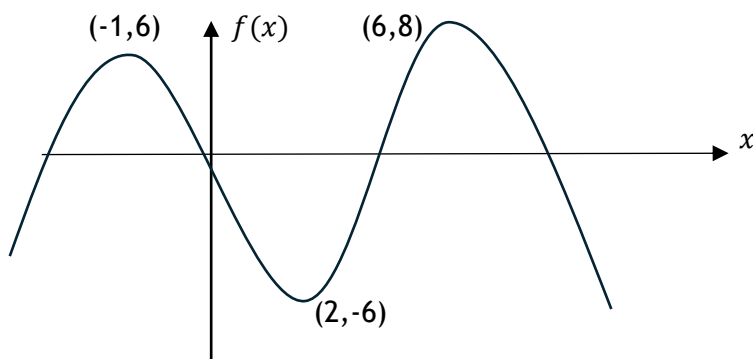
$$AB = 6 \text{ cm} ; CD = \sqrt{2} \text{ cm} ; AC = \sqrt{6} \text{ cm}.$$



Show that the exact value of $\cos(A + B)$ is $\frac{1}{\sqrt{3}}$

50. Shown below is part of the graph with equation $y = f(x)$.

Make a neat sketch of $y = f'(x)$.



51. Express $3x^2 - 24x + 17$ in the form $a(x + b)^2 + c$.

52. The straight line with equation $y = 7x - 27$ cuts the curve with equation $y = x^2 - 4x + 1$ at the points A and B. Establish the coordinates of A and B.

53. Evaluate

$$\int_{\frac{\pi}{3}}^{\pi} \sin 2x - \cos x \, dx$$

54. A loch has 24 tonnes of pollution.

An anti-pollutant is added weekly and reduces this level by 79%. At the end of each week, a further 8 tonnes of pollution is added.

The council aim to have the level of pollution in the loch no more than 9 tonnes in the long term. Will this level of anti-pollutant be enough to reach this level? Justify your answer.

55. For a function f :

- There is a repeated root at $x = 5$
- A stationary point exists at $x = 1$
- $f(-2) = 0$
- $f'(0) > 0$

Make a neat and annotated sketch of $y = f(x)$.

56a. Show that

$$3 - 4 \cos^2 x = 1 - 2 \cos 2x$$

56b. Hence find

$$\int 6 - 8 \cos^2 x \, dx$$

57. Establish the coordinates of the stationary points of the function $f(x) = x^3 - 3x^2 - 12$ and determine their nature.

58. A dosage of medicine in mg/l is administered in a patient's bloodstream. The rate at which it works through the bloodstream is modelled on the formula

$$P_t = 350e^{kt}$$

where P_t is the dosage remaining after t minutes.

t is the time in minutes since the dosage was administered

k is a constant.

- What dosage of medicine was initially administered?
- After 30 minutes, 210 mg/l remains. Work out the value of k .

59. Two vectors, p and q are defined as $p = \begin{pmatrix} 4i \\ 2j \\ -k \end{pmatrix}$ and $q = \begin{pmatrix} 3i \\ -2j \\ -5k \end{pmatrix}$.

Find the angle between these two vectors.

60. A curve, for which $\frac{dy}{dx} = 3x^2 - 8x$, intercepts a second curve at (2, -1).

Express y in terms of x .

61. Two points, A and B, have coordinates A(-3,8) and B(7,-2). Find the equation of the perpendicular bisector of AB.

62. Determine whether the points P(-4,1), Q(2,6) and R(3,9) are collinear.

63. Solve

$$3 \sin 2x + 4 \cos x = 0 \quad 0 \leq x \leq 2\pi$$

64. Explain why the sequence $u_{n+1} = 0.7u_n - 12$ has a limit and calculate this limit.

65. A function has equation $f(x) = (3 - 4x)^4$. Work out $f'(1)$.

66. Solve

$$\log_2(3x - 2) = 4$$

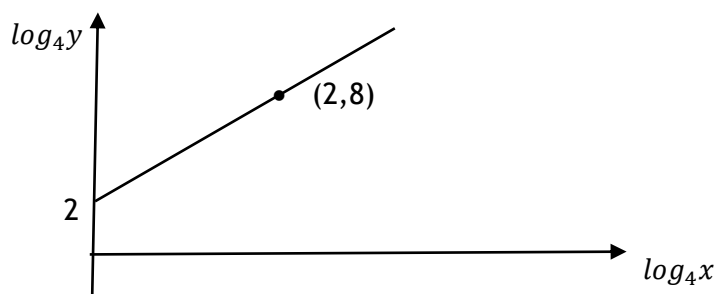
67. A straight line makes an angle of 135° with the positive direction of the x axis and passes through the point (-3,1). Find the equation of this line.

68. Show that $(x - 2)$ is a factor of $2x^4 + x^3 - 3x^2 - 10x - 8$ and factorise fully.

69. For what range of values of p does the equation $2x^2 + 2px + 5p = 0$ have no real roots?

70. Two variables, x and y , are connected by the equation $y = ax^b$.

The graph of $\log_4 y$ against $\log_4 x$ is shown.



Determine the values of a and b .

71. Show that $(x - 4)$ is a factor of $3x^3 - 17x^2 + 23x - 12$ and state clearly why there are no other linear factors.

72. State whether the function with equation $f(x) = 2x^4 + 5x^2 + 14x$ is increasing or decreasing at the point where $x = -1$.

73a. Show clearly that

$$(\sin x + 1)(2 \sin x - 1) = \sin x - \cos 2x$$

73b. Hence find

$$\int (\sin x + 1)(2 \sin x - 1) dx$$

74. Show that the curves with equations $y = 2x^2 + 9x + 6$ and $y = x^2 + x - 10$ touch at a single point and find the point of contact.

75a. A function f has equation $f(x) = 2x^2 - 1$. Find its inverse function, $f^{-1}(x)$, stating a suitable domain.

75b. Hence, sketch the graph of $y = f^{-1}(x)$

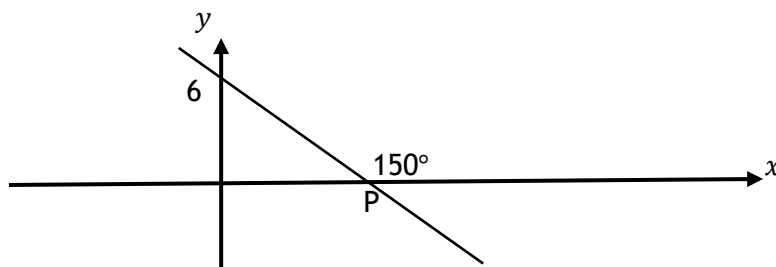
76 Solve

$$2 \cos 2x + \sqrt{3} = 0 \quad 0 \leq x \leq 2\pi$$

77. Evaluate

$$\log_2 12 - \frac{1}{3} \log_2 27$$

78. Find the equation of the straight line shown below and the exact coordinates of P, the point where it crosses the x axis.



79. A fleet of aircrafts is being manufactured. The value of the aircraft is based on the model

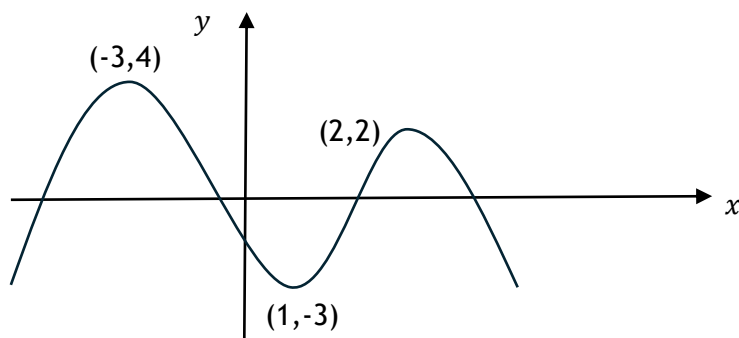
$$V(t) = 2.56e^{-kt}$$

where $V(t)$ is the value of the aircraft in £million after t months.

- a) What is the value of the aircraft initially?
- b) The aircraft decreases by 20% after 25 months. Work out the value of k .

80. Express $5 \cos x^\circ + 2 \sin x^\circ$ in the form $k \sin(x - a)^\circ$, stating its maximum and minimum value and the corresponding values of x at which these occur for $0^\circ \leq x \leq 360^\circ$

81. Shown below is the graph of $y = f(x)$.



- a) Sketch the graph of $y = f'(x)$.
- b) Hence, sketch the graph of $y = f'(x - 2) + 1$.

82. $(x + p)$ is a factor of the function with equation $f(x) = 2x^3 + 3x^2 - 11x - 6$.

- a) Find the value of p where $p < 0$.
- b) Hence fully factorise and solve $f(x) = 0$.

83. A has coordinates $(3, -7, 5)$ and B has coordinates $(-4, 0, 5)$.

P divides the line AB in the ratio 4:3.

Find the coordinates of P.

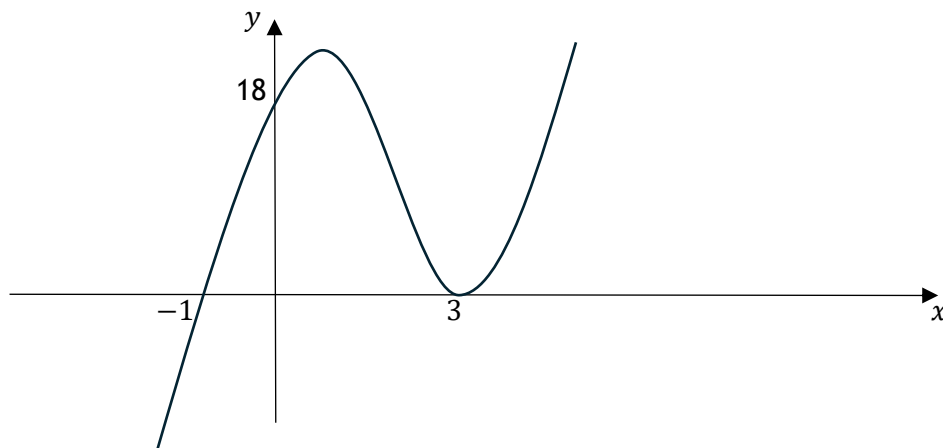
84. Two vectors, $\mathbf{a}$ and $\mathbf{b}$ are represented by $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$.

Calculate the angle between vectors $\mathbf{a}$ and $\mathbf{b}$.

85. Solve

$$2 \sin 2x^\circ - \cos x^\circ = 0 \quad 0^\circ < x < 360^\circ$$

86. The graph of $y = a(x + b)(x + c)^2$ is shown below. State the values of a, b and c .



87. A function has equation $f(x) = px^2 + 2px + 2$, $p \neq 0$.
For what value of p does $f(x)$ have equal roots?

88. State the maximum and minimum values of the function f with equation

$$f(x) = x^3 + 3x^2 - 9x - 17 \quad \text{for } -3 \leq x \leq 3.$$

89. Two functions, f and g exist where $f(x) = x^2 + 2x - 3$ and $g(x) = x + 3$.
A third function, h , also exists, where $h(x) = f(g(x))$.
For what range of values does the function h have no real roots?

90. Solve

$$\cos 2x + 6 \cos x + 5 = 0 \quad 0 \leq x \leq 2\pi$$

91. Show that $x = 4$ is the only **real** root of the equation $y = 2x^3 - 11x^2 + 17x - 20$

92. Differentiate

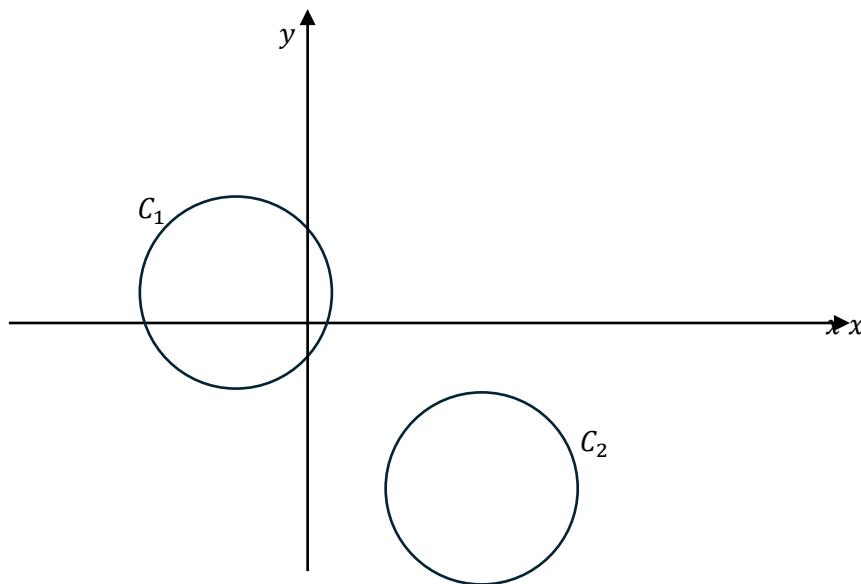
$$\frac{3}{\sqrt{(2x-5)}} \quad x > \frac{5}{2}$$

93. Express $4x^2 + 16x - 7$ in the form $a(x + b)^2 + c$.

94. Two identical circles, C_1 and C_2 are shown below.

Circle C_1 has equation $x^2 + y^2 + 4x - 8y - 29 = 0$

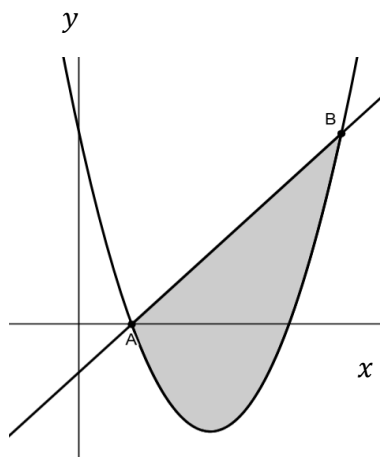
Circle C_2 has equation $x^2 + y^2 - 20x + 24y + c = 0$.



a) Show clearly that $c = 195$.

b) Two circles touch C_1 and C_2 externally. Find the equation of the larger circle.

95. The graph of $y = x - 1$ intersects with the graph $y = x^2 - 5x + 4$ at the points A and B as shown below.

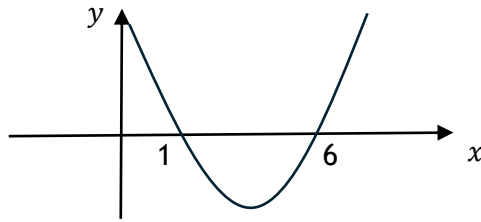


a) State the x coordinates of the points A and B.

b) Hence, find the area bound by the straight line and the curve.

96. Prove that the roots of the equation $x^2 + (1 - 2k)x - k = 0$ are real for all values of k .

97. The graph of $y = f'(x)$ is shown below. Stationary points occur when $y = -5$ and $y = 2$.



Draw a neat possible sketch of the graph of $y = f(x)$, annotating where possible.

98. Evaluate

$$\int_6^9 \frac{2}{\sqrt{(x-5)}} dx \quad x > 5$$

99. The coordinates of P, Q and R are given by $P(1,5,-1)$, $Q(0,6,2)$ and $R(4,3,4)$.

a) Express $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ in component form.

b i) Calculate $\overrightarrow{PQ} \cdot \overrightarrow{PR}$

ii) Hence calculate the size of angle QPR.

100. The graph of $y = a \cos bx + c$ is shown below.

Determine the values of a , b and c .

