

Higher Maths

300 Booster questions (Questions 201-300)

201. Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 8y - 5 = 0$ at the point $(-2, -1)$.

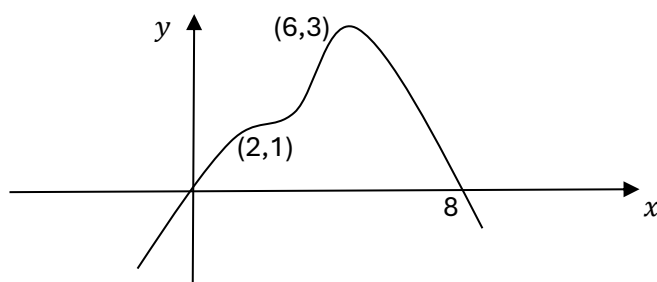
202. Solve

$$\log_3(x + 2) = 2\log_3 x + 1 \quad x > 0$$

203. Solve

$$\cos 2x - 3 \cos x - 1 = 0 \quad 0 \leq x \leq 2\pi$$

204. The graph below shows $y = f(x)$.



Sketch the graphs of:

a) $y = f(2x)$

b) i) $y = f'(x)$ ii) $f'(2x)$

205. Express $3x^2 - 18x + 11$ in the form $a(x + b)^2 + c$.

206. Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + x - 1$ where $x = 2$.

207. A straight line passes through the point $(-3, 2)$ and makes an angle of $\frac{2\pi}{3}$ radians with the positive direction of the x -axis.

Establish the equation of this line.

208. The number of homes using some form of renewable energy is modelled by

$$R = 7.12e^{0.0241t}$$

where R is the number of homes, in millions, using renewable energy t years after 2023.

a. How many homes were using renewable energy initially?

b. How long will it take to increase the number of homes using renewable energy by 10%?

209. A function has equation

$$f(x) = \sqrt{(3x + 7)^3}$$

Determine $f'(3)$.

210. Circle C_1 has equation $x^2 + y^2 - 6x - 2y - 6 = 0$.

Circle C_2 has equation $(x + 4)^2 + (y - 1)^2 = 9$.

a. Show that both circles touch externally.

b. Hence, find the coordinate of P, the point of intersection of circle C_1 and C_2 .

211. R is the point $(-2, 7, 6)$ and S is the point $(-7, 2, 1)$.

T divides RS in the ratio 2:3.

State the coordinates of the point T.

212. A function f is defined on $f(x) = 3x^2 - 4$.

Find a formula for the inverse function, $f^{-1}(x)$ and state a suitable domain for this inverse.

213. Determine

$$\int_1^4 2\sqrt{x} + \frac{3}{\sqrt{x}} dx.$$

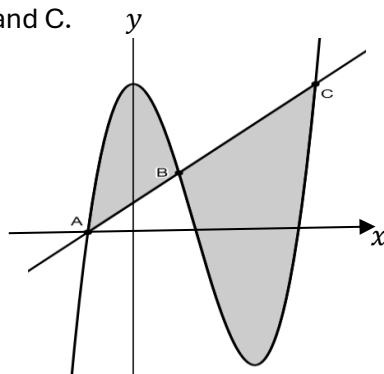
214. Vectors $a = -5i + xj - k$ and $b = xi + 2xj + 3k$ are perpendicular.

Determine the value of x where $x > 0$.

215. Given that $y = \frac{3x^3}{2\sqrt{x}}$ $x > 0$, find an expression for $\frac{dy}{dx}$.

216. Solve $\tan 3x + \sqrt{3} = 0$ $0 \leq x \leq \pi$

217. The diagram below shows the graph $y = x^3 - 4x^2 + 5$ intersecting with the line $y = x + 1$ at the points A, B and C.



The point A lies on the x axis.

- i State the coordinates of the point A.
- ii Determine the coordinates of B and C.

b. Hence, find the total shaded area.

218a. Express $3 \sin x^\circ - \cos x^\circ$ in the form $k \cos(x + a)^\circ$ where $0^\circ \leq a \leq 360^\circ$

b. Hence, state the minimum value of $6 \sin x^\circ - 2 \cos x^\circ$ and the corresponding value of x at which this minimum value occurs.

219. Two points, A and B, have coordinates $A(-2, 7)$ and $B(6, -5)$.

Find the equation of the perpendicular bisector of AB.

220. Simplify

$$\log_a ab^2 + \log_a a^2 b$$

221. Solve

$$\cos 2x + 3 \cos x - 1 = 0 \quad 0 \leq x \leq 2\pi$$

222. Find the equation of the tangent to the curve $y = x^3 - 5x^2 + x + 3$ where $x = 1$.

223. A butterfly kingdom has been reducing in size and a breeding program has been put in place. Currently, there is an estimated 14 000 butterflies in the population. Each month, 23% of this population is killed due to predators and environmental factors. However, another 1100 are added due to a careful breeding process and nature.

- Establish a recurrence relation for this and determine the number of butterflies after two months.
- In the long term, if this population falls below 4500, it will be in danger of becoming extinct. Is this population safe from extinction? Show your working.

224. Differentiate

$$\frac{3x^3 + 5x^2 - 2}{\sqrt{x}}$$

225. Three points with coordinates P(3,1), Q(4,4) and R(1,5) pass through circle C_1 .

- Show that PQ and QR are perpendicular.
- Find the midpoint of PR and hence state the equation of the circle C_1 .
- Circle C_2 has equation $x^2 + y^2 + 18x - 2y + 2 = 0$.
Show that circles C_1 and C_2 touch externally.

226. Two vectors have components $\mathbf{a} = \begin{pmatrix} 2p^2 + 1 \\ 5p - 3 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} p - 3 \\ p - 4 \\ p + 7 \end{pmatrix}$.

- Find an expression for $\mathbf{a} \cdot \mathbf{b}$ in terms of p .
- Show that when $p = 4$, vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
 - Hence establish the other value(s) of p for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

227. Triangle ABC has coordinates A(-6,4), B(8,6) and C(4,10).

- Find the equation of the median from C.
- Find the equation of the perpendicular bisector of BC.
- The median from C and the perpendicular bisector of BC intersect at the point P.
Establish the coordinates of P.

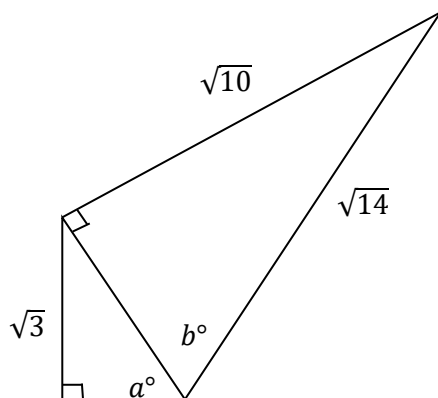
228. Solve

$$3 \sin 2x^\circ - 2 \cos x^\circ = 0 \quad 0^\circ \leq x \leq 360^\circ$$

229. The gradient of a curve is given by $\frac{dy}{dx} = 6x + 1$. It is known that the curve passes through the point (-2,7).

State the equation of the curve.

230. In the diagram below, show that the exact value of $\sin(a + b) = \frac{2\sqrt{3} + \sqrt{10}}{2\sqrt{14}}$

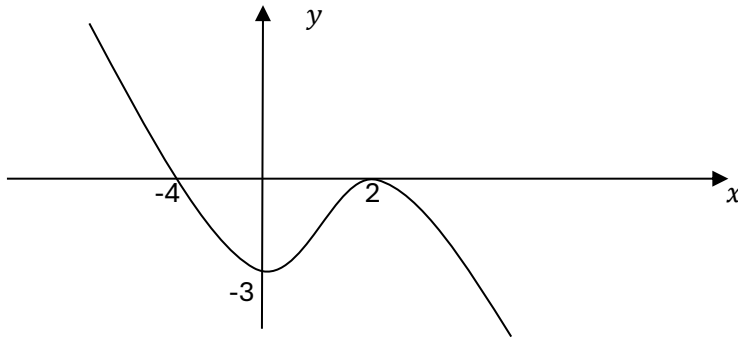


231. A straight line makes an angle of 60° with the positive direction of the x axis and passes through the point A(-2,5).

a. Establish the equation of this line.

b. B has coordinates (6,-3). Find the equation of the perpendicular bisector of AB.

232. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f'(x)$.



233. A function f has equation $f(x) = 2x^2 - 5$.

Find an expression for the inverse function $f^{-1}(x)$ and state a suitable domain.

234. Evaluate

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x + 4 \sin 4x \, dx$$

235. For what range of values of t does the equation $\frac{1}{2}tx^2 + (t-6)x + 4 = 0$ have no real roots?

236. Since 2021, referral numbers to a local hospital are modelled using the formula

$$N = 6400e^{-0.00673t}$$

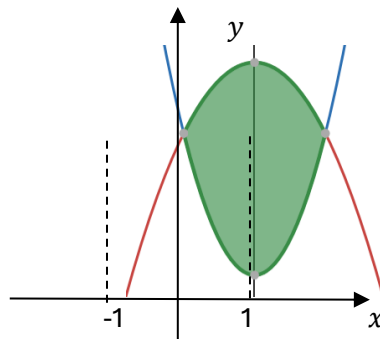
where N is the number of referrals after t months.

a. How many referrals were there at the start of 2021?

b. After how many months will the number of referrals reduce by one fifth?

237. The diagram below shows the graphs of $y = 6 - x^2$ and $y = 2x^2 + 3$ intersecting at the points where $x = -1$ and $x = 1$.

Calculate the shaded area.



238. Express $3x^2 - 18x + 11$ in the form $a(x+b)^2 + c$

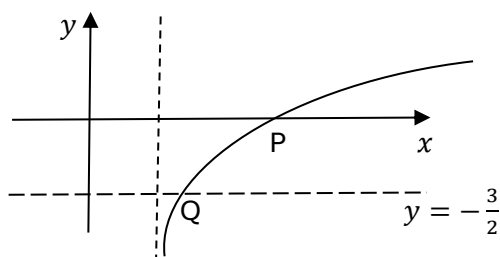
239. Two functions f and g are defined as

$$f(x) = (x + 3)$$

$$g(x) = 2x^2 - 5x$$

Establish an expression for $h(x)$ where $h(x) = g(f(x))$ and show that $h(x)$ has two real and distinct roots.

240. The graph of $y = \log_4(x - 2)$ is shown below. It crosses the x axis at the point P.



a. State the coordinates of the point P.

b. The graph of $y = \log_4(x - 2)$ intersects with the line $y = -\frac{3}{2}$ at the point Q.
Find the x coordinate of Q.

241. Find the rate of change of the function $f(x) = (3x + 7)^6$

242 a. Express $2 \cos x^\circ - 7 \sin x^\circ$ in the form $k \cos(x + a)^\circ$

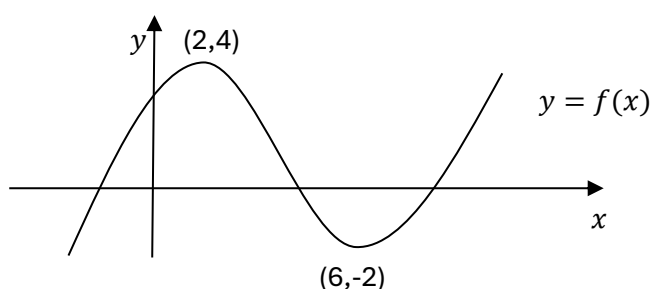
b. Hence, solve $2 \cos x^\circ - 7 \sin x^\circ + 5 = 0$ for $0^\circ < x < 360^\circ$.

243. Determine

$$\int \frac{2}{3\sqrt{x}} + 2x^2 \, dx$$

244. Find the equation of the tangent to the curve $y = 2x^3 - 4x^2 + 7x + 2$ at the point where $x = 2$.

245. The diagram below shows the graph of $y = f(x)$.



a. Sketch a diagram of $f'(x)$

b. On a separate diagram, sketch the graph of $y = 3 - f(2x)$.

246. The rate of change of the curve $y = 2 \sin(2x + a)^\circ$ is 2 where $x = 0$ and a is a constant.

a. Establish an equation for the rate of change and determine the value of a .

b. For what other value(s) of a , for $0^\circ \leq a \leq 360^\circ$, does this curve have the same rate of change?

247. Show that the line $y = 5x - 5$ is a tangent to the circle $(x - 7)^2 + (y - 4)^2 = 26$ and state the point of contact.

248. Differentiate

$$f(x) = \sqrt{\frac{x^3}{4}}$$

and evaluate $f'(4)$.

249. Find the equation of the line which passes through the point $(-3, -2)$ and makes an angle of $\frac{5\pi}{6}$ with the positive direction of the x axis.

250. The graphs of $y = 2 \cos 2x^\circ$ and $y = 4 \cos x^\circ + 1$ intersect at the points P and Q. Establish the x coordinate of P and Q.

251. Determine the integer value of p for which the equation $px^2 + (3p + 2)x + p = 0$, where $x \neq 0$, has no real roots.

252. Two points, A and B, have coordinates $(-4, 2, 8)$ and $(10, 2, 1)$ respectively. M divides the line AB in the ratio 3:4. Determine the coordinates of the point M.

253. A function has equation $f(x) = 2x^2 - 3, x \neq 0$. Establish an equation for the inverse function $f^{-1}(x)$ and hence state the expression for $f(f^{-1}(x))$.

254. The rate of change of a graph is given by the equation

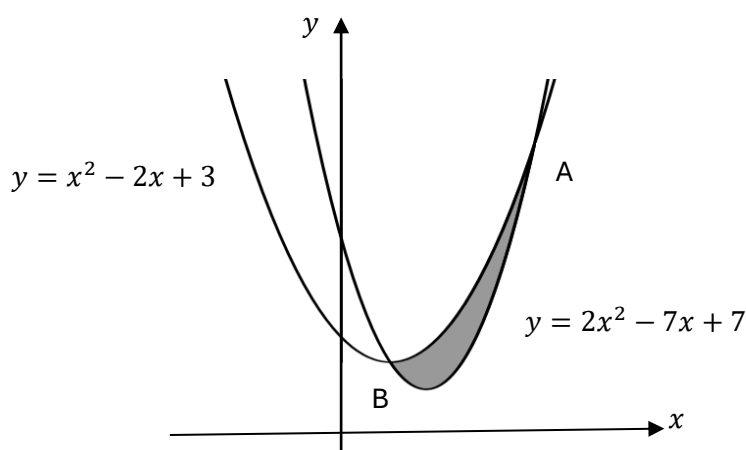
$$\frac{dy}{dx} = ax$$

where a is a constant.

It is known the graph intersects with a second graph at the points $(1, 5)$ and $(-2, 14)$.

Express y in terms of x .

255. In the diagram below, the graphs of $y = 2x^2 - 7x + 7$ and $y = x^2 - 2x + 3$ intersect at the points A and B.



- Establish the x coordinate of points A and B.
- Calculate the shaded area.

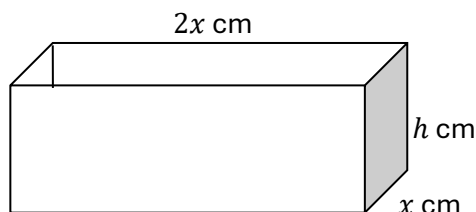
256. A graph has equation $f(x) = 2x^3 - 6x^2 + 1$.

- Establish the coordinates of the stationary points.
- State the maximum and minimum value of the graph for $-1 \leq x \leq 3$

257. Simplify

$$\log_9 3 + \frac{1}{4} \log_9 81 - \log_9 27$$

258. An open top box with dimensions $2x$ cm in length, x cm in breadth and h cm in height is shown in the diagram below.



The volume of the box is 36000cm^3

a. Show that the surface area of the box is given by

$$A(x) = 2x^2 + \frac{108000}{x}$$

b. Calculate the dimensions of the box which minimises this surface area.

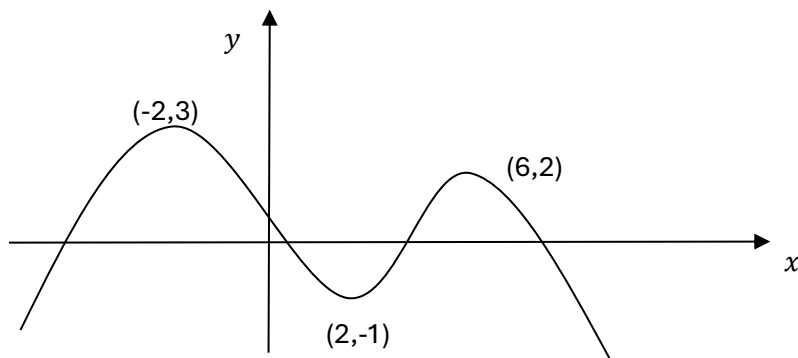
259. Find the equation of the straight line perpendicular to the line $\sqrt{3}x - y + 2 = 0$ and passing through the point $(2,3)$.

260. Show that $x = 2$ is the only real root of $y = 3x^3 - 11x^2 + 14x - 8$.

261. a. Show that $(\cos^4 x - \sin^4 x)(\cos^2 x + \sin^2 x) = \cos 2x$.

b. Hence find $\int_{\frac{\pi}{3}}^{\pi} 9 \cos^4 x - 9 \sin^4 x \, dx$

262. Shown below is the graph of $y = f(x)$.



On separate diagrams, neatly sketch graphs of

a) $y = f\left(\frac{1}{2}x\right) + 2$ b) $y = f'(x)$ c) $y = f'(x - 3)$

263. Find the equation of the tangent to the circle $x^2 + y^2 - 6x + 8y - 40 = 0$ at the point $(-1,3)$.

264. Solve $2 \cos 2x^\circ + 5 \cos x^\circ - 7 = 0$ $0^\circ \leq x \leq 360^\circ$

265. Determine whether the function $f(x) = 2x^3 + 5x^2 + 1$ is increasing or decreasing at $x = -2$.

266. A circle has equation $x^2 + y^2 - 8x - 2y - 3 = 0$.

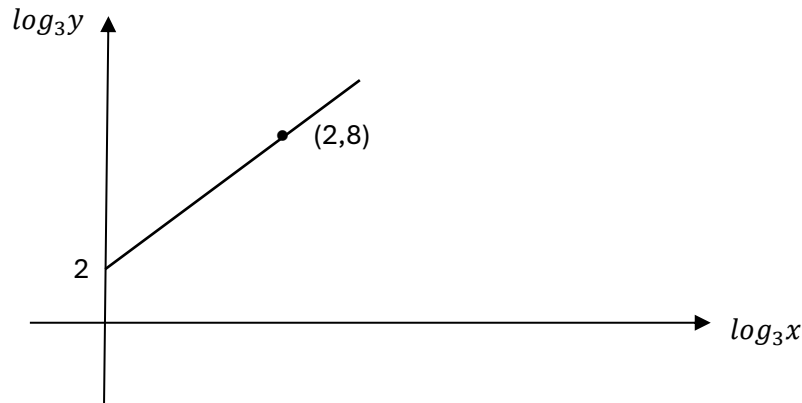
- Determine the equation of the tangent at the point (8,3).
- A second tangent is drawn at the point (2,5). Find the equation of this tangent and hence establish the coordinates of the point of contact.

267. Find the rate of change for the function $f(x) = \sin^3 x^\circ$ when $x = 60^\circ$.

268. Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$.

269. Two variables are connected by the equation $y = ax^b$.

The graph of $\log_3 y$ against $\log_3 x$ is a straight line as show below.



Find the values of a and b .

270. A recurrence relation exists such that $u_{n+1} = 0.75u_n + 6$.

- Explain why this sequence had a limit and calculate this limit.
- A second recurrence relation exists such that $u_{n+1} = au_n + 9$ and has the same limit. Calculate the value of a .

271. A function has equation $f(x) = 4 \cos x + \sin x$.

- Express $f(x)$ in the form $k \sin(x + a)^\circ$.
- Hence solve $f(x) + 2 = 0$ $0^\circ \leq x \leq 360^\circ$

272. Show that

$$\frac{\tan 60^\circ}{\sin 45^\circ} - \frac{\cos 30^\circ}{\sin 210^\circ} = \sqrt{6} + \sqrt{3}$$

273. Two functions, f and g , are defined as $f(x) = 4x - 1$ and $g(x) = x^2 - 2$.

A third function, h , is defined as $h(x) = \frac{1}{f(g(x))}$.

Find an expression for $h(x)$ and state a suitable domain.

274. Triangle ABC has coordinates A(8,2), B(0,6) and C(2,-2).

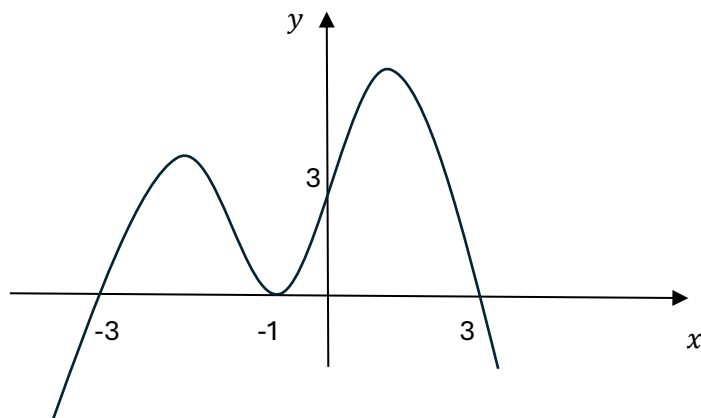
- Find the equation of the median from A.
- Find the equation of the altitude from C.
- Find the point of intersection of this median and altitude.

275. Vectors $\mathbf{a}$ and $\mathbf{b}$ have components $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$.

- Calculate $\mathbf{a} \cdot \mathbf{b}$
- Calculate the size of the angle between vectors $\mathbf{a}$ and $\mathbf{b}$.

276. The diagram below shows the graph with equation $y = k(x - a)(x - b)(x - c)^2$.

State the values of k , a , b and c .



277. Find the range of values of p for which the equation $x^2 + (p - 3)x + 4p = 0$ has no real roots.

278. The gradient of a curve is defined as $\frac{dy}{dx} = 6x - 3$.

Its graph passes through the centre of a circle with equation $x^2 + y^2 - 2x + 8y - 3 = 0$.

Express the equation of the curve in terms of x and y .

279a. Establish the x coordinates of the stationary points of the curve with equation

$f(x) = 2x^3 + 3x^2 + 3$ and determine their nature.

- Hence, find the maximum and minimum value of the function between $-2 \leq x \leq 1$.

280. Circle C_1 has equation $(x - 7)^2 + (y - 3)^2 = 16$ with centre P.

Circle C_2 has equation $x^2 + y^2 - 4x - 16y + 43 = 0$ with centre Q.

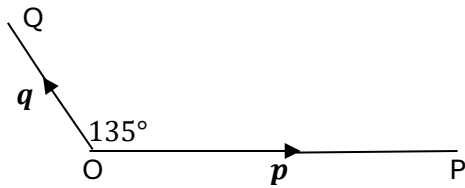
- State the centre and radius of circles C_1 and C_2 .
- Show clearly that the circles intersect.
- The line PQ is drawn. M divides PQ in the ratio 3:2.
 - Establish the coordinates of M.
 - Show that M lies inside only one of these circles.

281. The straight line with equation $y = a - 2x$ is a tangent to the curve $y = x^2 + 2x + 2$.
Determine the value of a .

282. Solve

$$\cos 2x^\circ - \sin x^\circ + 2 = 0 \quad 0^\circ \leq x \leq 360^\circ$$

283. The vector diagram below shows $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$



If $|\overrightarrow{OP}| = 2\sqrt{3}$ and $|\overrightarrow{OQ}| = \frac{3}{4}|\overrightarrow{OP}|$, show that $\mathbf{p} \cdot \mathbf{q} = -\frac{9\sqrt{2}}{2}$

284. Find the rate of change of $f(x) = \frac{3x^3 - 9x^2}{2x\sqrt{x}}$ when $x = 4$.

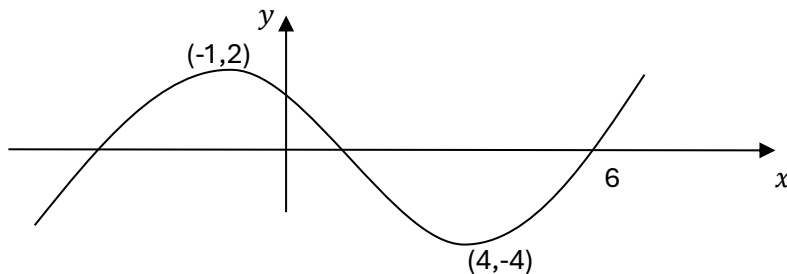
285. a. Show that the equation $2\log_2 x + \log_2(3x + 2) = 5$, $x > 0$ has a solution at $x = 2$.
b. Show clearly there are no other solutions.

286. Find $\int \frac{2}{\sqrt[3]{(2x+7)^5}} dx$

287. The line with equation $y = 2x - 1$ intersects the circle with equation $x^2 + y^2 - 4x - 6y - 7 = 0$ at the points A and B. State the coordinates of A and B.

288. Determine whether the function $f(x) = \frac{5}{2\sqrt[3]{x}}$ is increasing or decreasing when $x = -1$.

289. The graph of $y = 4 - 2f(x)$ is shown below. Sketch and annotate the graph of $y = f(x)$.



290. Given that $\sin A = \frac{\sqrt{10}}{4}$, $0^\circ < A < 90^\circ$, find the exact value of:

a. $\sin 2A$

b. $\cos 4A$

291. Show that the line with equation $y = 2x + 5$ does not intersect the circle with equation $x^2 + y^2 - 6x - 4y - 2 = 0$.

292. Establish the value of p where

$$\int_p^{3p} (x + 4) dx = 32$$

293. Circle C_1 has equation $x^2 + y^2 - 4x + 10y - 7 = 0$.

Circle C_2 has equation $(x - 2)^2 + (y + 2)^2 = 9$.

Determine whether these circles intersect, touch – internally or externally - or do not intersect.

294. Two museum exhibits have been destroyed in a fire.

An oil painting, initially worth £3100, is losing 11% of its value every year, but restoration is increasing its value by £260 per year.

a. Write a recurrence relation to represent this model.

A clay sculpture, initially worth £2300, is losing 19% of its value, but restoration is increasing its value by £320.

b. Write a recurrence relation to represent this model.

c. The museum decides that, if it is to maintain restoration costs, each exhibit should return to at least 75% of its original value.

Should the museum decide to maintain restoration of either exhibit? Justify your answer.

295. A circle passes through the points A(-1,-1), B(7,3) and C(-5,7).

a. Find the equations of the perpendicular bisectors of AB and AC.

b. Hence or otherwise, establish the equation of the circle passing through all 3 points.

296. Two functions, f and g , have equations

- $f(x) = x^3 + 6x^2 + 5x - 2$
- $g(x) = 2x^2 + 8$

Show algebraically that f and g intersect at only one point and state the point of contact.

297. A function has equation $f(x) = \frac{2x^4+5x}{x\sqrt{x}}$.

Find the value of $f'(4)$.

298. a. Show that $(x + 3)$ is a factor of $2x^3 - 3x^2 - 23x + 12$ and fully factorise.

b. Hence, or otherwise, solve

$$\log_3 \frac{2x^3-3x^2-23x+12}{x^2-x-12} = 2$$

299. The gradient of a function is expressed by

$$\frac{dy}{dx} = 4x - 6$$

When $x = 5$, $y = 40$.

Express y in terms of x .

300. Delta Eggs have a new logo designed.

The logo consists of

- The curves with equations $y = x^2$ and $y = 8 - x^2$
- Triangle ABD where AB has equation $x = -1$.
- Horizontal line symmetry along CD.

Calculate the shaded area of the logo.

